Problem Set - 3

* problems can be considered simpler than others.

Deadline:-23rd September, 2006 by night 11 P.M

1) Find all pairs m, n of positive integers such that $m^2 - n$ divides $m + n^2$ and $n^2 - m$ divides $m^2 + n$.

2) A, B, C is a triangle. X, Y, Z lie on the sides BC, CA, AB respectively, so that AYZ and XYZ are equilateral. BY and CZ meet at K. Prove that $YZ^2 = YK \cdot YB$.

3) ABCD is a parallelogram. K is the circumcircle of ABD. The lines BC and CD meet K again at E and F. Show that the circumcenter of CEF lies on K.

4) *a1, a2, a3, ... an is a sequence of non-zero integers such that the sum of any 7 consecutive terms is positive and the sum of any 11 consecutive terms is negative. What is the largest possible value for n?

5) *Find $a_1^3/(1 - 3a_1 + 3a_1^2) + a_2^3/(1 - 3a_2 + 3a_2^2) + \dots + a_{101}^3/(1 - 3a_{101} + 3a_{101}^2)$, where $a_n = n/101$.

6) Show that (36m + n)(m + 36n) is not a power of 2 for any positive integers m, n.