I speak to you, the language of numbers,
Elementary, yet profound,
We're all not sure how it works,
But it works! The approach must be sound!

I was hungry the other day, so were more than three of my comrades,
Apples, we thought! Apples will muffle the stomach's growls!
Three random groups of it we formed, a, b, c,
A trickster came along and did something that fooled us alls!

To each bundle he added a particular k when we were not looking,
And we couldn't divide any of the bundles amongst us!
So alas, from eating the apples we abstained,
And the integers a, b, c and k remained.

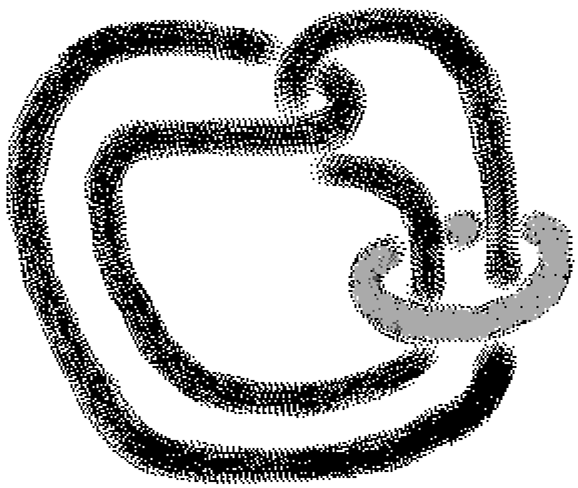
A map I found, of times long gone,
From [0,1]X[0,1] to [0,1],
We considered this F (a, x), how shall we define it?
To **x** you add the remainder of dividing **a** by 1, that messes up things quite a bit!

The map threw its puzzle; I in turn pose it you,
Perhaps you will have more luck at it, at least more than I do.

When a is irrational and $0<\zeta<1$, there exists non-negative integers m, n such that either $x_0 + na - m = \zeta$, in which case one and only one member of $\{x_n\}$ equals ζ, or $x_0 + na - m \neq \zeta$ for all m, $n \geq 0$ and there exists an infinite subsequence $\{n_r\}$ of integers such that $0 \leq n1 < n2$….. and $x_{nr} \rightarrow \zeta$ as $r \rightarrow \infty$.

For your sake, I hope the case is as the first,
It'll ease things up in good measure,
For when $x_n=\zeta$,
You have reached the treasure!

This is all too high-headed, who finds a treasure like this?
I understand your chagrin; Columbus would not have wrestled so,
To find new land, or to find the Indians,
Perhaps we need something that we can 'feel' more?



There are two threads, the one in black and the other in gray,
Do your magic, interchange the two,
Threads cannot be broken, only deformed,
They cannot be twisted or coloured into each other too!
Child's play maybe? Much too easy,
But playing with the child in you can get quite crazy.

Enough of your long-winded sentences, I thought this was a contest,
On math and skill, not on poetry and rhyme!
I agree; the gravity of the situation weighs upon me,
As does your short attention span. I waste no more of your time.

4. a) The sequence $a_1, a_2, a_3 \ldots$ of positive integers is determined by its first two members and the rule $a_{n+2} = (a_{n+1} + a_n)/\gcd(a_n, a_{n+1})$. For which values of a_1 and a_2 is it bounded?

b) $p(x)$ is a quadratic polynomial with non-negative coefficients. Show that $p(xy)^2 \leq p(x^2)p(y^2)$.

5. Let σ be a non-singular curve of the second degree and A_1, A_2, A_3, A_4, A_5 and A_6 be points on it. Then the three points where the straight lines A_1A_5 and A_2A_4; A_3A_4 and A_1A_6; A_2A_6 and A_3A_5 meet are on the same straight line.

One last problem, one last rhyme,
My finer sentiments itch to be expressed,
Actions speak louder than words they say,
But this is an online event! My actions are suppressed!

Geometry never goes out of fashion,
It is in this spirit that I beg you to draw a circle with center O,
Draw a diameter AB, then a perpendicular chord CD,
Then a chord AE, wait! There's more.

AE goes through the mid-point of radius OC,
Dear god! What does this imply?
That DE goes through the mid-point of chord BC! Not amazed?
Perhaps you know enough to tell me exactly why.

The show is over, I pack my bags,
To ponder over the questions, I leave you in peace,
For those who have survived the tripe so far and want the problem set plainly,
I give it below to put you to ease.

1. Prove that if a, b, c are any integers, & n is an integer>3, then there exists an integer k such that none of the numbers k+a, k+b, k+c is divisible by n.

2. Consider the map F: [0,1]X[0,1] → [0,1] such that

$$F(a, x) = x + a \text{ modulo } 1$$

Note that $x_n = F^n(a, x_0)$. When a is irrational and $0<\zeta<1$, there exists non-negative integers m, n such that either $x_0 + na - m = \zeta$, in which case one and only one member of $\{x_n\}$ equals ζ, or $x_0 + na - m \neq \zeta$ for all m, n ≥0 and there exists an infinite subsequence $\{n_r\}$ of integers such that $0 \leq n1 < n2 \ldots$.. and $x_{nr} \to \zeta$ as $r \to \infty$. Prove this result.

3. The union of disjoint knots is called a link. Find a deformation for the link shown in the picture above that interchanges the two components.

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b) p(x) is a quadratic polynomial with non-negative coefficients. Show that $p(xy)^2 \leq p(x^2)p(y^2)$.

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6. The chord CD of a circle center O is perpendicular to the diameter AB. The chord AE goes through the midpoint of the radius OC. Prove that the chord DE goes through the midpoint of the chord BC